# FREE SPACE MONITORING SYSTEM FOR MOTOR VEHICLES

## Background of the Invention

### Field of Invention

**[0001]** The invention concerns a process and device for monitoring the free space in the direction of travel of a vehicle according to the precharacterizing portion of Patent Claims 1 and 10.

### Related Art of the Invention

**[0002]** In complex vehicle maneuvers it is often difficult for the driver of the motor vehicle to predict the precise track of the vehicle and to know how much free space will be needed in order to avoid collision. This is the case in particular when the vehicle operator is not familiar with the dimensions or the maneuvering characteristics of the vehicle.

**[0003]** In order to support the vehicle operator during difficult driving maneuvers, an operator information system has been developed as described in Japanese Published Application JP 2000-67395 A, wherein image data describing the roadway have predicted vehicle tracks superimposed thereupon. In this manner it becomes possible for the vehicle operator to better estimate the future behavior of his vehicle. To do this the system uses the data of two sensors, which are mounted on the front wheels of the vehicle and therewith provide information regarding the direction of motion and the speed of the vehicle. From the sensor data a future vehicle track is predicted and this is superimposed as a line structure of a depiction of the vehicle track in a display unit. A disadvantage with such a system is that the predicted vehicle track is superimposed only upon a

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display of the vehicle roadway or route, with no images embedded
in the display providing the operator information regarding
objects in the vicinity of the vehicle.

[0004]    In order to provide the vehicle operator an improved
overview over the actual environment of the vehicle and to allow
better maneuvering therein, an improved environment monitoring
system is proposed in Patent Application EP 1 158 803 A2.
Herein the situation of the environment of the vehicle is
detected using multiple camera systems, with data transformed
into a birdseye view.    Into this image a vehicle system
representing the vehicle is embedded in a position corresponding
to the vehicle position.    The vehicle symbol is herein so
designed, that it fits into the environment image with its
correct size relationship.    The image information provided to
the vehicle operator thus corresponds to the view from the point
above the vehicle looking down onto the environment; that is,
the vehicle and its environment are represented in a type of
birdseye perspective.    In order to support the vehicle operator
in his evaluation of the future travel path of the vehicle, the
image has superimposed thereon a three dimensional line contour
describing the predicted vehicle path of travel.    The line
contour represents the image of an edge or frame of a rectangle,
which is curved along the predicted path of travel.    The two
lower longitudinal edges of the rectangle lie on the vehicle
path of travel and correspond substantially to the vehicle track
represnted in JP 2000-67395 A.    The height and breadth of the
rectangle is herein selected to be smaller than height and
breadth of the vehicle.    The vehicle operator obtains by this
mode of representation on the one hand a good depiction of the

future relationship of the vehicle to its environment; however, due to the perspective of representation which is unconventional for the vehicle operator it is not possible for him, in simple manner to grasp the danger of collision, which would be attributable to the parts projecting from the vehicle sides into the area of the predicted vehicle path of travel. In particular, the parts projecting into the predicted vehicle path from above can only be represented and recognized with difficulty. It is also not possible to represent for the vehicle operator thoroughfares (tunnels or entryways, garages) in a plausible or intuitively understandable manner.

## SUMMARY OF THE INVENTION

[0005]    It is thus the task of the invention to provide a process and a device which makes possible for the operator of a vehicle to envision the future vehicle path of travel with regard to its environment, and to reliably recognize dangers of collision due to parts projecting from the side or upwards into the predicted vehicle path of travel.

[0006]    This task is solved by a process and a device with the characteristics of Patent Claims 1 and 10.

[0007]    Advantageous embodiments and further developments of the invention can be seen from the dependent claims.

[0008]    In a process for monitoring the environment space in the direction of travel of the own vehicle, image data of the environment around the vehicle in the direction of travel of the vehicle is recorded using a camera system. Further, the three

dimensional free space required for unimpeded travel is calculated in advance in a signal processing unit on the basis of the operating parameters and the dimensions of the vehicle. The operator of the motor vehicle thus has displayed to him in a display at least those parts of the image data obtained from the camera system and corresponding to the area of the required free space. In inventive manner, the image data are herein displayed substantially from a perspective which would represent the environment in the path of travel of the vehicle from the direct perspective of the operator of the vehicle. Further, in inventive manner the image data associated with the required free space is subjected to a further processing, whereby as a result of this further processing, the vehicle operator is informed regarding whether or not a sufficient free space is available to him for unimpeded travel. It is thus possible therewith, in inventive manner, by a continuous evaluation of the image data, to automatically react to a dynamic change in the vehicle environment and to inform the vehicle operator as to whether a sufficient free space is still available or not for unimpeded travel. By the display of at least the part of the image data representing the required free space, the vehicle operator has the possibility of independently assessing and evaluating the potential of danger in the predicted path of travel. The display of the image data occurs therein not in the perspective of the representation according to the systems known in the state of the art, in which information is presented as viewed from above (birdseye view), but rather essentially from the perspective wherein the environment around the vehicle would appear to the vehicle operator as it would in his own direct view in the direction of travel. It is herein however also

conceivable, that the depiction of the vehicle environment, in particular as dictated by the optics of the camera system (for example, wide-angle lens or catadiopter camera lens), is displayed distorted. On the other hand it is however also very conceivable, that in the framework of the further data processing any warping or distortion of the image data is corrected. In a manner which is understandable to him, the vehicle operator sees an image depicting the environment, of which the perspective substantially corresponds with that of his own view, as he would himself perceive by directly looking out of the window. It is thus no longer necessary that the vehicle operator take the unaccustomed birdseye view perspective representation and to convert it into the real conditions.

[0009]  Data regarding the dimensions of the vehicle are stored in a memory, and is accessible to the signal processing unit, in order, with the aid of the operating parameters of the vehicle, to be able to estimate the necessary driving space for the driving maneuver. Therein there is the possibility that the dimensions stored in the memory do not correspond with the actual dimensions of the vehicle, but rather are expanded by a certain safety margin. One such safety margin could in advantageous manner be up to 20 centimeters. It is also conceivable that this safety margin is manipulated by the signal processing unit to be variable and, depending upon the vehicle operating speed and/or danger potential, that it becomes enlarged or reduced. It is also possible that the safety margins are variously adjusted for the various areas or parts of the vehicle. Thus it is for example conceivable that the safety

distance from bumpers is set smaller than that for the contact-sensitive rear-view mirrors.

[00010]  In inventive manner, this memory for recording the vehicle dimensions is so arranged and organized, that therein there are stored a multiplicity of data sets having different vehicle dimensions, which can be individually selected by a targeted selection, and provided to the signal processing. The memory can also be provided with a means, by which at least one of the data sets stored in the memory can be changed out. This provides the advantage, that the system for monitoring environmental free space is relatively easy to adapt to changes (for example in the vehicle contour, e.g. new trailer, different vehicle superstructure, etc.). Precisely such a design is of great advantage above all in the case of operation of a fleet of vehicles, in particular trucks with variable superstructures. In this way even electro-technologically unsophisticated operating personnel can operate the inventive system using simple input elements. They can do this without change of electro-technological components or connecting with or operating programming devices.

[00011]  In particularly preferred manner these changes in the dimensions of the vehicle can also be assumed or incorporated in simple manner in the memory associated with the signal processing device, in that the data set describing the new dimensions are read into the memory in simple manner via a mobile or stationary data transmission unit using a suitable interface. In particularly advantageous manner, in the case of a suitable design of the system the inputting and the selection

of data sets can also occur wirelessly, via a communication device. In the case of frequent repetitive (identical) changes (for example driving with loaded or as the case may be unloaded container) it also possible that multiple various vehicle contours or dimensions are stored in memory of the data processing unit, and can be called up at will. In particular, a keypad accessible by the vehicle operator can be provided for this purpose. In particularly simple and useful manner, this keypad has individual keys associated with the data sets stored in memory, so that with a single key stroke selection the desired data set can be called up.

## Brief Description of the Drawings

[00012]    In the following the invention will be described in greater detail on the basis of the illustrated embodiment shown in the figures.

Fig. 1    shows an image scene (10) in which the edge borders of the free space are visualized by superimposition of walls represented in symbolic form (16).

Fig. 2    shows an image scene (20) identical to Fig. 1, wherein here the edge borders of the free space are visualized by superimposition of a winding symbolic represented virtual driving tube (26) with checkerboard pattern.

Fig. 3    shows the relationship of the global coordinate system to the vehicle coordinate system.

Fig. 4    shows the image geometry of a parabolic mirror.

Fig. 5       shows the relationship between the coordinates of an
             omnicam and those of a pinhole camera.

Fig. 6       shows the inverse transformation of the image position
             of an omnicam with reference to a pinhole camera
             model.

Fig. 7       shows the image recorded by an omnicam in the
             direction of travel, in which the horizon is
             illustrated.

Fig. 8       shows the optical flow field of the image points
             within the scene.

Fig. 9       shows the optical flow field for a wall, which rises
             perpendicular to the optical axis of the omnicam, in
             the case of movement along the optical axis.

Fig. 10      shows the optical flow of a sidewall during the
             movement of the vehicle.

Fig. 11      shows the optical flow of a horizontal plane of the
             movement of the vehicle.

Fig. 12      shows the depth map of a corridor.

Fig. 13      shows the optical flow for the corridor of Fig. 12.

Fig. 14    shows,    color-encoded,    the    flow    field    for    the
           horizontal flow shown in Fig. 13.

Fig. 15    shows, color-encoded, the flow field for the vertical
           flow shown in Fig. 13.

## Detailed Description of the Invention

[00013]    After the path of travel and the therefore required
free space has been predicted by the signal processing unit, it
is particularly advantageous when in the framework of the
further processing of the image data any dynamic time-based
changes of this data are taken into consideration.   In this
manner the system can recognize objects or persons which enter
into the free space required for the vehicle maneuver.   If this
is the case, then the operator of the vehicle should be informed
in an appropriate manner that sufficient free is no longer
available.   This information can occur in particular by optical,
acoustic or haptic perceivable signal-providers.   On the other
hand, it is also conceivable that the image data displayed to
the vehicle operator of objects or persons representing an
impediment in the free space would be represented visually in
enhanced or accentuated form.   For this, there is suitable for
example a color enhancement or false color accentuation of the
display, in particular a false-color representation, increasing
in steps depending upon the danger of collision (for example:
high collision danger → object is colored red; slight danger of
collision  →  object  is  colored  yellow).    In  particularly
preferred  manner  the  objects  or  persons  representing  an
impediment could also be highlighted in the manner that symbols
are superimposed thereon.   For example, in the framework of the

further processing of the image data, objects which have moved are detected by means of an image processing algorithm based upon the optical flow, then in suitable manner the moved object can have superimposed symbolic movement vectors (arrows) which describe the direction of movement and movement speed (length of the arrow).

[00014] A particularly advantageous mode and manner of informing the vehicle operator regarding the amount of free space available to him is comprised therein, that in the framework of the further processing of the image data, the image data displayed to the vehicle operator has superimposed a symbolic representation of the boundary limits of the required free space as predicted in the signal processing unit. Therein it is particularly advantageous to describe the edge boundaries by walls represented symbolically. The height of the walls corresponds therein to the height of the required free space (that is, essentially the dimensions stored in memory regarding the maximum height of the vehicle).

[00015] In particularly advantageous manner the edge boundaries are so represented that the required free space is bordered by a type of "driving tube" or "driving tunnel"; for this, the edge boundaries occur by symbolic represented walls and the overhead area in the manner of a tube. By this "tube view" the attention of the vehicle operator is specifically directed, and it also becomes possible for him with greater ease to substantially recognize objects which project into the needed free space from above or from the sides.

[00016]   For the driver, the estimation of whether spatially fixed objects extend into the rearward free space becomes particularly simple when, during the (backwards) travel, he can compare the relative movement of objects in the image (display) with the relative movement of a spatially fixed symbolic driving tube: if the objects move faster than the boundaries of the driving tube, then they are spatially closer - that is - within the direct area of danger.   If during travel they move slower through the image, then they are outside of the driving tube and there is no danger that they will be contacted during further driving.   For this reason it is advantageous to so design the walls and overhead, and in certain cases even the floor, of the symbolic shown tube, such that it

1.)   has a geometrically clear and distinct identifiable structure (for example: grid pattern, checkerboard pattern or series of columns)

2.)   provides a "through view" to the video scene lying therebehind (for example: broken away or semi-transparent structures of the walls and/or the ceiling).

3.)   is respectively so calculated or, as the case may be, determined based on the vehicle movement, that it remains stationary in the longitudinal direction.

[00017]   In order to make the superimposition of the camera image with the symbolic depiction of the edge boundaries of the required free space appear as realistic as possible, the image characteristics (transformation equations) of the camera image

are to be taken into consideration. For a better understanding, in particular with regard to the realization of the advantageous design introduced above under point 3 (standing still in the longitudinal direction) of the graphic overlay, in the following there will be discussed as an example the necessary transformation equations for the superposition of an image recorded by means of a catadiopteric camera (omnicam) with a parabolic mirror. Therein, the basic steps and, in detail, the individual steps, of the transformation equation are explained and elaborated, which are necessary for the inventive superimposition of the camera image with the edge boundaries of the required free space.

[00018] Of course the invention is not limited to the employment of such a camera, but rather necessary transformations explained on the basis of this camera geometry in detail allow themselves to be directly applied in simple manner to other camera systems with other mapping or transformation characteristics.

**Coordinate Systems**

[00019] The coordinate systems used herein follow the conventions from the literature regarding optical flow. In the 3D-camera coordinate system the Z-axis indicates the direction of view, the X-axis to the right and the Y-axis downwards. The center image coordinates are provided by an x-axis towards the right and a y-axis directed downwards. The origin of both coordinate systems is the center of the image.

**Regarding Environment-to-Vehicle Coordinates**

**[00020]**  For a camera mounted on a moving vehicle, one can first introduce two important coordinate systems: an environment coordinate system, in which the vehicle environment exists, and a vehicle coordinate system, which moves along with the vehicle, see Fig. 3.  When the vehicle moves in the direction of the $Z_w$-axis, the transformation between these two coordinate systems is described by the position W(t) of the vehicle in the environment coordinate system:

$$\begin{pmatrix} X_V \\ Y_V \\ Z_V \end{pmatrix} = \begin{pmatrix} X_W \\ Y_W \\ Z_W \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ W(t) \end{pmatrix} \qquad (1)$$

**[00021]**  The instantaneous speed of the vehicle is V = W. Therewith, the time derivation of the point in which the vehicle environment rests is:

$$\begin{pmatrix} X_V \\ Y_V \\ Z_V \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -V \end{pmatrix} \qquad (2)$$

**Regarding Vehicle-to-Camera Coordinates**

**[00022]**  It is shown in Fig. 3 that the direction of view of the camera is tilted about an angle $\tau$ against the (backwards) direction of movement.  In this case the transformation of vehicle to camera coordinates occurs as follows:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\tau & -\sin\tau \\ 0 & \sin\tau & \cos\tau \end{pmatrix} \cdot \begin{pmatrix} X_V \\ Y_V \\ Z_V \end{pmatrix} \qquad (3)$$

$$\begin{pmatrix} X_V \\ Y_V \\ Z_V \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\tau & \sin\tau \\ 0 & -\sin\tau & \cos\tau \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \qquad (4)$$

[00023]    In the general case, these transformations contain not only the tilt angle $\tau$, but rather the three Euler angles.

[00024]    From Equation 2 one obtains the movement of a stationary point in the camera coordinates

$$\dot{X} = 0 \qquad\qquad\qquad (5)$$

$$\dot{Y} = V \sin \tau \qquad\qquad (6)$$

$$\dot{Z} = -V \cos \tau \qquad\qquad (7)$$

**Polar Camera Coordinates**

[00025]    The arctan $(y,x)$ – function with two arguments determines the phase of the complex number $x + iy$ in the value range $[-\pi, \pi]$.    The sequence of the arguments were selected following the standard or paradigm of the ANSI-C function *atan2*.

[00026]    Transformation of polar into Cartesian coordinates:

$$X = R \sin \theta \cos \phi \qquad\qquad (8)$$

$$Y = R \sin \theta \cos \phi \qquad\qquad (9)$$

$$Z = R \cos \theta \qquad\qquad\qquad (10)$$

**[00027]**    Transformation of Cartesian into polar coordinates:

$$R = \sqrt{X^2 + Y^2 + Z^2} \tag{11}$$

$$\theta = \arctan\left(\sqrt{X^2 + Y^2}, Z\right) \tag{12}$$

$$\phi = \arctan\left(Y, X\right) \tag{13}$$

## Polar Image Coordinates

**[00028]**    Transformation of polar into Cartesian coordinates:

$$x \quad = \quad r \cos \phi \tag{14}$$

$$y \quad = \quad r \sin \phi \tag{15}$$

**[00029]**    Transformation of Cartesian into polar coordinates:

$$r = \sqrt{x^2 + y^2} \tag{16}$$

$$\phi = \arctan(y, x) \tag{17}$$

## Point of Intersection of Direction of View with Plane of Object Space

**[00030]**    The vector of the direction of view $w$ is provided in the vehicle coordinates.    In the case $w_1 \neq 0$, the beam cuts through or intersects the plane $X_V = a$ at the point

$$p = \frac{a}{w_1} w \tag{18}$$

**[00031]**    In the case $w_2 \neq 0$, the beam intersects the plane $Y_V = b$ at the point

$$p = \frac{b}{w_2} w \qquad\qquad (19)$$

[00032]    In the case $w3 \neq 0$, the beam intersects the plane $Z_v = c$ at the point

$$p = \frac{c}{w_3} w \qquad\qquad (20)$$

**Omnicam Transformation for Image or Mapping or Picture Mirror Form**

[00033]    The transformation through the omnicam modeled here occurs in two steps: a reflection in a parabolic mirror occurs from an orthographic image.

[00034]    The parabolic mirror surface with a focal length $f$ is defined by

$$z = f - \frac{x^2 + y^2}{4f} . \qquad\qquad (21)$$

[00035]    This expression is used in many of the following equations. That means, in those cases $z$ is for the term on the right side of the equation, which depends from the position $(x, y)$. Since the focus of the parabola originated in coordinate data, the direction of view at the image point $(x, y)$ is defined by

$$s\ (x,y)\ =\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ f - \dfrac{x^2 + y^2}{4f} \end{pmatrix} \qquad (22)$$

[00036]    The derivation of the parabolic equation (21) is then

$$\frac{dz}{dx}\ =\ -\ \frac{x}{2f} \qquad (23)$$

$$\frac{dz}{dy}\ =\ -\ \frac{y}{2f} \qquad (24)$$

[00037]    That means, the normal vector on the mirror surface has the direction

$$m\ (x,y)\ =\ \begin{pmatrix} x \\ y \\ 2f \end{pmatrix} \qquad (25)$$

[00038]    Therein it is not postulated or claimed that $s$(x, y) or $m$(x, y) are unity vectors.

**Projection of Points**

[00039]    The omnicam image is represented most simply in polar coordinates:

$$r\ =\ 2f\ \tan\ \frac{\theta}{2} \qquad (26)$$

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**[00040]** As discussed above, $\tau$ is the distance from the image center, measured in pixels. $\theta$ is the angle of the image direction to the optical axis. This relationship can be derived with the aid of Fig. 4 and Equation (25). The azimuth $\phi$ is the same in the camera and image coordinates.

**[00041]** In Cartesian coordinates this means

$$x = \frac{2fX}{R+Z} = \frac{2fX}{\sqrt{X^2+Y^2+Z^2}+Z} \tag{27}$$

$$y = \frac{2fY}{R+Z} = \frac{2fY}{\sqrt{X^2+Y^2+Z^2}+Z} \tag{28}$$

**Projection of Lines**

**[00042]** A straight line in the space is defined by a point $p$ and a direction $l = (l_1, l_2, l_3)^t$ with $|l| = 1$. The vector $n = p$ x $l$ is perpendicular to the plane, which is spanned by the straight line and origin of the camera coordinates. The camera components of $n$ determine the type of the omnicam image of the 3D-lines:

- in the case $n = 0$, the line cuts through or intersects the origin and its image is a point with the coordinates

$$x = \frac{2fl_1}{1+l_3} \tag{29}$$

$$y = \frac{2fl_2}{1+l_3} \qquad (30)$$

- in the case $n \neq 0$, however $n_3 = 0$, then the 3D-line parallel to the optical axis and its image is a 2D-line through the image center with the equation

$$n_1 x + n_2 y = 0 \qquad (31)$$

- in the other case, the image of the line is a arch or circle segment with the center point $(x_c.y_c)$ with a radius $r$:

$$x_c = 2fn_1/n_3 \qquad (32)$$
$$y_c = 2fn_2/n_3 \qquad (33)$$
$$r = 2f|n_1|/n_3 \qquad (34)$$

**Distance from Plane**

[00043] The length of the vector $\boldsymbol{s}(x,y)$ from Equation 1 (22) is

$$|s(x,y)| = \sqrt{x^2+y^2+z^2} = f + \frac{x^2+y^2}{4f} = 2f - z \qquad (35)$$

[00044]   Thus, a point with the distance $R$ with the image position (x,y) has the camera coordinates

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R \ \frac{s(x,y)}{|s(x,y)|} \tag{36}$$

[00045]   This produces a compact relationship between the camera coordinates of the object point on the one hand as well as distance $R$ an image position (x,y) on the other hand:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{R}{2f-z} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{37}$$

or in greater detail

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{R}{4f^2 + x^2 + y^2} \begin{pmatrix} 4fx \\ 4fy \\ 4f^2 - x^2 - y^2 \end{pmatrix} \tag{38}$$

[00046]   The distance of a perpendicular wall with the equation X = a is provided for an image point (x,y) by

$$R(x,y) = \frac{2f-z}{x} \ a \tag{39}$$

or in greater detail

$$R(x,y) = \frac{4f^2 + x^2 + y^2}{4fx} \, a \qquad (40)$$

[00047]    In the case $a < 0$ the wall is visible only on the left side of the image ($x < 0$), and the reverse.  A horizontal plane has the equation $Y_V = b$ in the environment coordinate system. With equation (4) the camera coordinates can be transformed

$$Y \cos \tau + Z \sin \tau = b \qquad (41)$$

[00048]    Introduced into equation (38):

$$R(x,y) = \frac{2f - z}{y \cos \tau + z \sin \tau} \, b \qquad (42)$$

or in greater detail

$$R(x,y) = \frac{4f^2 x^2 + y^2}{4fy \cos \tau + \left(4f^2 - x^2 - y^2\right)\sin \tau} \, b \qquad (43)$$

[00049]    In the case $b > 0$, the plane is visible only below the horizon.

**Reflection of Omnicam Images**

[00050]    For omnicams with an effective projection point, the omnicam-image can be reconstructed from a pinhole camera

perspective. [1] The centered pinhole camera coordinates are provided by the projection equation

$$x' = f \frac{X}{Z}$$

(44)

$$y' = f \frac{Y}{Z}$$

(45)

[00051]   The relationship between the omnicam and the pinhole camera coordinates are provided by a compression function, which can be derived from equation (27f):

$$\kappa(\rho) = \frac{2}{1+\sqrt{1+\rho^2}}$$

(46)

[00052]   A graph of this compression function can be seen in Fig. 5.

[00053]   The omnicam image position (x,y) corresponds to the centered pinhole camera coordinates (x',y') according to

$$x = \kappa\left(\frac{\sqrt{x'^2+y'^2}}{f}\right) x'$$

(47)

$$y = \kappa\left(\frac{\sqrt{x'^2+y'^2}}{f}\right) y'$$

(48)

[00054]    The inverse transformation is

$$x' = \frac{4f^2}{4f^2 - x^2 - y^2}\, x \tag{49}$$

$$y' = \frac{4f^2}{4f^2 - x^2 - y^2}\, y \tag{50}$$

[00055]    A graph of this coefficient $4/(4 - \rho^2)$ is shown in Fig. 6.

## Horizon and FOE

[00056]    For describing the effect of the vehicle movement in the image one uses the term *horizon* and *expansion point* (or FOE for *focus of expansion*).   Fig. 3 shows how a tilted camera is moved along by the vehicle, which moves along a horizontal plane.   The horizon is the image of a plane parallel thereto, which passes through the projection center, as shown in Fig. 7. The main point of the camera is marked with "x", the FOE with a "+", the horizon has a yellow line.

[00057]    A straight line in the horizontal plane is provided by a point $p_V = (0,0,1)^t$ and the direction $l_V = (1,0,0)^t$.   A normal or perpendicular in the horizontal plane is $n_V = (0,0,1)^t$, and has the camera coordinates $n = (0,\ \cos\ \tau,\ \sin\ \tau)^t$.   As discussed above, one must distinguish between the two cases.

•    for camera tilt $\tau = 0$, the horizon is a straight line with $y = 0$.

- for $\tau \neq 0$, the horizon is a circular arc of which the center point and radius are defined by

$$x_H \quad = \quad 0 \tag{51}$$

$$y_H \quad = \quad 2f \cot \tau \tag{52}$$

$$r_H \quad = \quad 2f / \sin \tau \tag{53}$$

[00058]   For a straight linear horizontal movement shown in Fig. 3 the FOE lies in the center of the horizon:

$$x_{FOE} \quad = \quad 0 \tag{54}$$

[00059]   The ordinate is determined for $\tau \neq 0$

$$y_{FOA} \quad = \quad y_H \; - \; r_H \; = \; -2f \; \tan \frac{\tau}{2} \tag{55}$$

as follows also from Formula (26).

[00060]   In a symbolic representation of the free space by a grid pattern it is precisely the grid elements which are comprised of perpendicular and horizontal segments of straight lines $p_i$, $l_i$, according to the discussion regarding Formula (29) through (34), wherein the $p_i$ represents the model point-of-space points on the straight line segments, and $l_i$ respectively represent one of two unity vectors in the direction of the corresponding grid elements.  These straight line segments are, as discussed, transformed into circular arc segments on the image sensor surface and these are represented as grid patterns. The locationally fixed representation of the symbolic grid elements are then accomplished thereby, that in the case of a

movement along the segment s in the longitudinal direction of the vehicle - measured for example by a rotary encoder or a speed sensor at the wheel - all model points or space points $l_i$ are displaced or moved along the segment s opposite to the direction of travel.

$$p_i^{new} = p_i - s*e$$

wherein e represents a unity vector in the vehicle longitudinal direction in the camera coordinate system.

[00061]  It is further useful, that the driver need not continuously move his vehicle in order to determine a danger of objects in the described manner on the basis of the movement pattern in the image.  It should be made possible to him, for example by operating a switch, to generate a repetition of the last movement sequence on the display system.  This can for example be the image material of the last five-meter travel segment with "driving tube" included, shown in an endless loop.

[00062]  Possible design or presentation elements of the symbolic driving tube, which satisfy the requirements for the clear geometric structure and the transparency described in the last section are for example the representation of the edge boundaries by means of a grid pattern, a sequence of columns or a checkerboard pattern.

[00063]  Fig. 1 shows an image scene 10, in which the edge boundaries of the free space are visualized by superimposing symbolic represented walls 16.  The image data associated with

the rear scene are distorted with the advantageous employment of a catadiopoteric camera. For a vehicle operator however, the essential object in the environment, such as for example the vehicle **11** and **12** as well as the garage **13** with the roll-up door **14** is very easy to recognize and understand. In the shown example there is formed, by the use of the catadiopoteric camera, the part area **15** lying in the direction of travel of the own vehicle at the lower edge of the displayed or presented image data. Such a display has the advantage, that the vehicle operator is able to observe those areas of the vehicle having the greatest potential of collision danger.

[00064] Without superimposing upon the image data the symbolic walls **16** it would be difficult for the vehicle operator however to estimate whether he can drive into the garage **13** with his vehicle, without bumping against the partially lowered roll-up door **14**. The symbolic walls show him however that for his vehicle there is sufficient free space below the roll-up door, in order to drive into the garage without collision. From the straight forward progression of the symbolic walls, the vehicle operator can derive that his vehicle is optimally oriented for driving into the garage, and see that there is no longer any need for any essential steering correction during operating the vehicle.

[00065] Fig. 2 shows an image scene (20) identical to Fig. 1, wherein here the edge boundaries of the free space are visualized by superimposing a symbolic represented virtual driving tube (26) with checkerboard pattern. By the use of a checkerboard pattern for accentuating the edge boundaries, the

observation of the image scene on the display becomes clear, and the movement of the own vehicle is particularly understandable. This is particularly advantageous in the situation where driving must occur at slow speed due to close proximity.

[00066] By the use of clear geometric structures in the already discussed example of the symbolic driving tube, in combination with a registering lens, for example a catadiopoteric lens, a further advantage is achieved: by the reference structure of the symbolic driving tube displayed in the same manner, it becomes substantially easier for the driver to evaluate and take into consideration without having to think of the distortions of the lens. This is particularly conspicuous in the examples shown in Figs. 1 and 2, at the edge of the image to the right and left, in which the view onto a diagonal drive path is to be seen. For example, the warped boundaries of the rectangular driving tube shown in Fig. 2 facilitates the interpretation, as to which of the elements lying above and below are in the scene. In the distortion shown here the street lies "inside" as the lower element, and the sky lies at the outer edge of the circular-shaped representation.

[00067] In particularly advantageous manner the invention is suited for rear-space monitoring during a re-orienting or repositioning operation of a motor vehicle. Herein the rearward space is often not directly observable by the vehicle operator, so that he is dependent upon a particularly clear depiction of the environment and the interaction of his vehicle therewith.

[00068]  A further supporting and assisting of the driver during repositioning or re-orientation can be achieved, when the above described comparison of the movement of a locationally fixed driving tube with the actual movement of objects in the image are no longer left to him on the basis of how things appear, but rather are undertaken automatically by a computation unit.  This is possible with the utilization of, for example, processes known in robotics for the comparison of the measured optical flow in the image with the calculated optical flow of a model vector field, here the model of a geometically defined driving tube corresponding to Fig. 13.

[00069]  The algorithm necessary for the automatic evaluation and comparison of the optical flow is described in the following by way of example for the use of a camera system comprised of a catadiopoteric camera (omnicam) with a parabolic mirror.  Of course these basic principles can, in analogous manner, also be applied to other camera systems and other optical characteristics.

**Optical Flow**

**General Optical Flow**

[00070]  The derivative of the omnicam projection Equation (27f) is

$$\overset{\bullet}{x} = \frac{2f \cdot \left(R \cdot (R+Z) - X^2\right)}{R \cdot (R+Z)^2} \overset{\bullet}{X} - \frac{2fXY}{R \cdot (R+Z)^2} \overset{\bullet}{Y} - \frac{2fX}{R \cdot (R+Z)} \overset{\bullet}{Z} \quad (56)$$

$$\overset{\bullet}{y} = -\frac{2fXY}{R \cdot (R+Z)^2} \overset{\bullet}{X} + \frac{2f \cdot \left(R \cdot (R+Z) - Y^2\right)}{R \cdot (R+Z)^2} \overset{\bullet}{Y} - \frac{2fY}{R \cdot (R+Z)} \overset{\bullet}{Z} \quad (57)$$

[00071]    If one again introduces the projection Equation (27f),
there results

$$\dot{x} = \left(z+\frac{y^2}{2f}\right)\frac{\dot{X}}{R} - \frac{xy}{2f}\frac{\dot{Y}}{R} - x\frac{\dot{Z}}{R} \tag{58}$$

$$\dot{y} = -\frac{xy}{2f}\frac{\dot{X}}{R} + \left(z+\frac{x^2}{2f}\right)\frac{\dot{Y}}{R} - y\frac{\dot{Z}}{R} \tag{59}$$

**Optical Flow for Tilted Camera**

[00072]    For a tilted camera as in Fig. 3 the movement is
described by Equation (5f).    If one introduces this into the
Equation (58f) above, there results

$$\dot{x} = \frac{Vx}{R} \cdot \left(\cos\tau - \frac{y}{2f}\sin\tau\right) \tag{60}$$

$$\dot{y} = \frac{V}{R} \cdot \left(y\cos\tau + \left(z+\frac{x^2}{2f}\right)\sin\tau\right) \tag{61}$$

[00073]    Therein one must take into consideration that $R$ and $z$
depend upon the image position.    In greater detail, the vertical
flow says that

$$\dot{y} = \frac{V}{R} \cdot \left(y\cos\tau + \frac{4f^2+x^2-y^2}{4f}\sin\tau\right) \tag{62}$$

[00074]    The distance $R(\chi,\gamma)$ modulates the horizontal and
vertical flow with the same front or lead factor.    Thus $R(\chi,\gamma)$

influences only the length of the flow vectors, however not their direction. As shown, the direction of flow is determined only by the camera movement, that is, through the bearing or location of the point of expansion. For this translation movement the direction of the flow vectors is also independent of the depth structure of the environment.

## Optical Flow of the Camera in the Direction of Travel

[00075]    For an omnicam, of which the axis is oriented in the direction of travel, the flow Equation (60f) is simplified:

$$\overset{\bullet}{x} = \frac{Vx}{R} \tag{63}$$

$$\overset{\bullet}{y} = \frac{Vy}{R} \tag{64}$$

[00076]    This is very similar to the field of flow of the pinhole camera. The only difference is comprised in that the denominator $R$, in the case of the pinhole camera, would be $Z$. In both camera types the flow field is in this case purely radial, that is the flow vector $(\overset{\bullet}{x},\overset{\bullet}{y})$ is parallel to the radius vector $(x,y)$, also when $R(x,y)$ changes over the image. For a front parallel wall with a depth $Z = c$ there follows from Equation (38)

$$R(x,y) = \frac{4f^2 + x^2 + y^2}{4f^2 - x^2 - y^2} c \tag{65}$$

[00077]    Thus  the  corresponding  flow  field  appears  as  shown  in
Fig. 9:

$$\dot{x} = \frac{Vx}{c} \cdot \frac{4f^2 - x^2 - y^2}{4f^2 + x^2 + y^2} \tag{66}$$

$$\dot{y} = \frac{Vy}{c} \cdot \frac{4f^2 - x^2 - y^2}{4f^2 + x^2 + y^2} \tag{67}$$

## Optical Flow of a Vertical Side Wall

[00078]    One  then  takes  into  consideration  again  a  camera  with
tilt  angle  $\tau$  at  the  vehicle  as  in  Fig.  3.   The  vertical  side  wall
parallel  to  the  direction  of  travel  is  determined  by  X = a.   The
distance  of  the  wall  according  to  Equation  (39)  can  be  utilized
in  the  general  flow  Equation  (60f):

$$\dot{x} = \frac{V}{a} \cdot \left( \cos\tau - \frac{y}{2f} \sin\tau \right) \cdot \frac{x^2}{2f - z} \tag{68}$$

$$\dot{y} = \frac{V}{a} \cdot \left( y\cos\tau + \left( z + \frac{x^2}{2f} \right)\sin\tau \right) \cdot \frac{x}{2f - z} \tag{69}$$

[00079]    In  greater  detail,  this  means

$$\dot{x} = \frac{V}{a} \cdot \left( \cos\tau - \frac{y}{2f} \sin\tau \right) \cdot \frac{4fx^2}{4f^2 + x^2 + y^2} \tag{70}$$

$$\overset{\bullet}{y} = \frac{V}{a} \cdot \left( \cos\tau + \frac{4f^2 + x^2 - y^2}{4fy}\sin\tau \right) \cdot \frac{4fxy}{4f^2 + x^2 + y^2} \tag{71}$$

[00080]    This is illustrated in Fig. 10.

**Optical Flow of a Horizontal Plane**

[00081]    A horizontal plane (for example a street or a garage roof) is defined by $Y_V = b$.    The wall distance according to Equation (42) can be introduced into the general flow Equation (60f):

$$\overset{\bullet}{x} = \frac{Vx}{b} \cdot \left( \cos\tau - \frac{y}{2f}\sin\tau \right) \cdot \frac{y\cos\tau + z\sin\tau}{2f - z} \tag{72}$$

$$\overset{\bullet}{y} = \frac{V}{b} \cdot \left( y\cos\tau + \left(z + \frac{x^2}{2f}\right)\sin\tau \right) \cdot \frac{y\cos\tau + z\sin\tau}{2f - z} \tag{73}$$

$$\tag{74}$$

[00082]    In greater detail this thus means

$$\overset{\bullet}{x} = \frac{Vx}{b} \cdot \left( \cos\tau - \frac{y}{2f}\sin\tau \right) \cdot \frac{4fy\cos\tau + \left(4f^2 - x^2 - y^2\right)\sin\tau}{4f^2 + x^2 + y^2}$$

$$\overset{\bullet}{y} = \frac{Vy}{b} \cdot \left( \cos\tau + \frac{4f^2 + x^2 - y^2}{4fy}\sin\tau \right) \cdot \frac{4fy\cos\tau + \left(4f^2 - x^2 - y^2\right)\sin\tau}{4f^2 + x^2 + y^2}$$

**[00083]**   This is illustrated in Fig. 11.

**[00084]**   For simplification, one can employ the third binomial series:

$$\overset{\bullet}{x} = \frac{Vx}{b} \cdot \left(\cos\tau - \frac{y}{2f}\sin\tau\right) \cdot \frac{4fy\cos\tau + \left(4f^2 - x^2 - y^2\right)\sin\tau}{4f^2 + x^2 + y^2} \tag{75}$$

$$\overset{\bullet}{y} = \frac{V}{4fb} \cdot \frac{\left(4fy\cos\tau + \left(4f^2 - y^2\right)\sin\tau\right)^2 - x^4\sin^2\tau}{4f^2 + x^2 + y^2} \tag{76}$$

## Optical Flow of a Corridor

**[00085]**   Fig. 12 shows the depth map of a corridor. The side columns show the colors employed for encoding depth. The depth is calculated according to Equation (39) for the vertical walls and according to Equation (42) for the horizontal floor and the ceiling.

**[00086]**   Fig. 13 shows the theoretical expected flow for this corridor. It can also be calculated according to Equation (70f) for the vertical walls and Equation (75f) for the floor and ceiling. Horizon and FOE are marked as above as the yellow circular arc or, as the case may be, a plus sign. Figs. 14 and 15 show the same flow field as color encoded images respectively for the horizontal and vertical flow.

[00087]    Those locations in the image, in which the model flow is less then the actual measured flow, are potential collision candidates and are displayed as such to the vehicle operator in a noticeable or standout color, and in the case of sufficient magnitude can also be employed as trigger for generating a warning signal, for example an acoustic warning signal.